Mysterious Addables
&
Other Ponderables!

For the Young and Old

By Wizard John

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Notice Page

$$3^2+4^2=5^2$$

&

$$3^3+4^3+5^3=6^3$$

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Wright-Patterson Air Force Base, near Dayton, Ohio, is one of the nation's most important military installations. The base is headquarters for a vast, worldwide logistics system and is the foremost research and development center in the U.S. Air Force. Steeped in tradition, WPAFB has been a leader in military aviation development from the time of airplane inventors Wilbur and Orville Wright, who lived in Dayton, to today's aerospace age.

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Introduction

Pattern is a basic ingredient that we human beings seek to find in both the natural and social world. Neither disappoints us since both supply an endless source of information that can be linked and arranged in patterns. Traditionally, mathematics has been one of the primary tools used to explore patterns. This book focuses on patterns and problems that can be explored or solved using basic mathematics—from arithmetic through elementary algebra. Teachers of Grades 2 through 12 will find plenty of gems that can be used in the classroom both as enrichment and skill-building activities. All problems and activities are identified per the subject abbreviation table found on page 9. Where topics and/or problems fit one or more subject categories, subject abbreviations are separated by commas.

Everyone will enjoy something from within these pages! For example, children love magic squares, the Kaprekar process, and Lewis Carol's word morphing. Likewise, logic enthusiasts will find several new challenges awaiting them—such as "The Camel and the Bananas" on page 27. There is certainly enough "W.OW! Stuff" (and just a wee bit of "Wiz Stuff") to keep both young and old curled up in a chair or bent over a page—eager pencil in hand—for a long, long time.

Wizard John

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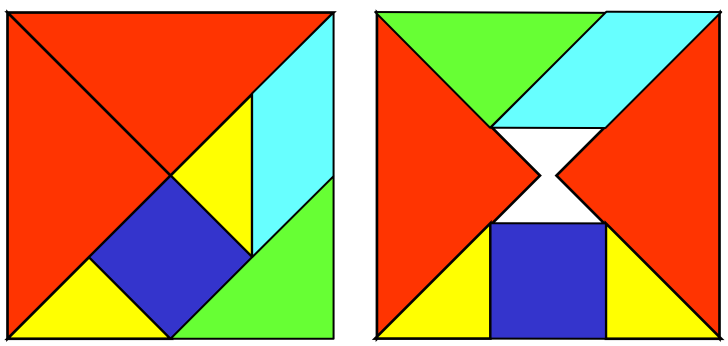
Subject Abbreviations

Used in "W.O.W! Stuff"

Subject	**Abbr.**
Logic	*LG*
Arithmetic	*AR*
Algebra	*AL*
Geometry	*GE*
Curio	*CU*
Langage	*LA*
Word Play	*WP*



The Tangram Area Paradox,
How is the Hole Created?

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Some W.O.W! Stuff

10	18	1	14	
11	24	7	20	3
17	5		21	9
23	6	19	2	15
4		25	8	16

A 5 by 5 Magic Square with Three Missing Numbers. Can you fill them in!

The Old Glory Puzzle *LG, GE*

September 11, 2001 is a date that we baby boomers will remember in much the same fashion that our parents remembered December 7, 1941. Our flag is once again enjoying a newfound popularity! Early baby boomers, such as myself, were born under a forty-eight star flag. This flag was arranged in six rows of eight stars each. Hawaii joined the Union in 1959, leading to a forty-nine star flag—seven rows of seven stars each. Alaska joined the Union one year later, leading to the present fifty star flag arranged in nine slightly nested rows alternating seven, six, seven, six, seven, six, seven, six, and seven stars.

Suppose new states are added to the Union during the current century. Possibilities might include Puerto Rico, Guam, and the District of Columbia.

Challenge: Arrange three rectangular fields to accommodate fifty-one, fifty-two, and fifty-three stars. Use the dual constraint that there shall be no more than nine rows and no more than eight stars per row, a historical precedent. Having problems? Step away from the rectangular pattern—literally, out-of-the-box thinking—and go to a circular pattern, utilized at least once in our nation's history.

Finally, for those of you who need even more of a challenge, keep on adding the states and stars all the way to seventy-two—nine times eight.

Twin Towers Numerology *CU, AR*

The following number curios were sent to me shortly after the World Trade Center bombing. We humans are always seeking meaning and significance in those tragic or happy events that affect our lives. The establishing of numerical patterns is one way (and a very ancient one) that people use to explore meaning.

- The date of the attack: 9/11 = 9+1+1 = 11
- September 11, 254th day of the year: 2+5+4 = 11
- After September 11, 111 days are left in the year
- 119 is the area code for Iran/Iraq: 1+1+9 = 11
- The Twin Towers looked like the number 11
- The first plane to hit the towers was Flight 11
- The 11th state added to the Union was New York
- New York City has 11 letters
- Afghanistan has 11 letters
- "The Pentagon" has 11 letters
- Ramzi Yousef has 11 letters (convicted in the World Trade Center bombing of 1993)
- Flight 11 had 92 people on board: 9+2 = 11
- Flight 77 had 65 on board: 6+5 = 11

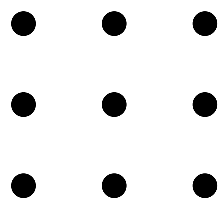
A question to ponder: Is there a significant event in your life where you have used numerical patterns (perhaps dates) to help establish meaning? Martin Gardner (of Scientific American fame) has given the name jiggery-pokery to such quests for numerological patterns. Jiggery-pokery or not, we will continue as rational beings to seek and establish patterns of all sorts.

II

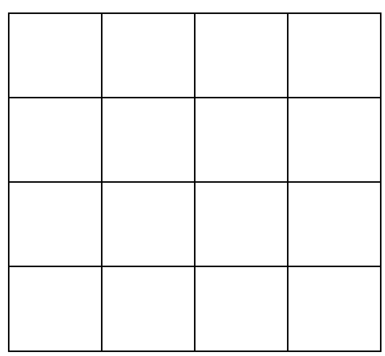
Two Squares and Two Challenges *LG, AR*

The following two puzzles have been used as icebreakers for years in various group settings. Both are simple yet profound and illustrate the use of out-of-the-box or lateral thinking.

1) Try to connect all nine dots using just four straight-line segments and one continuous pen stroke.



2) Count the total number of squares contained in the big square below.



Ten Commandments of Algebra *AL*

Algebra can be thought of as a language, universal in scope! Many people are frustrated when learning this language because they fail to follow a few basic study rules. Here are ten such rules written in yesterday's English.

1. Thou shall read thy problem.
2. Whatsoever thou shall do to one side of the equation, do thou also to the other side.
3. Thou shall draw a picture when thou tackles a word problem in order to actively engage both sides of thy brain.
4. Thou shall ignore the teachings of false prophets to do complicated work in thy head.
5. Thou must use thy "Common Sense", or else thou wilt have flagpoles 9000 feet in height, yea…even fathers younger than sons.
6. When thou does not know, thou shall look it up; and if thy search is fruitless, thou shall ask the teacher.
7. Thou shall master each step before putting down in haste thy heavy foot on the next.
8. Thy correct answer does not always prove that thou has understood or correctly worked the problem.
9. The shall first see that thou has copied thy problem correctly before bearing false witness that the book is a father of lies.
10. Thou shall look back even to thy youth and remember thy arithmetic.

☺ABCDF☹

Eight Ways to Dismantle 666 *AR, CU*

The number 666 has been in Western thought for about twenty centuries. 666 would have been easily written in Roman times as DCLXVI, a simple descending sequence of the first six Roman numerals. Today, writing 654321 would serve the same purpose. 666 has many fascinating numerical properties, eight of which are listed below.

1) $666 = 6 + 6 + 6 + 6^3 + 6^3 + 6^3$
2) $666 = 1^6 - 2^6 + 3^6$
3) $666 = 2^2 + 3^2 + 5^2 + 7^2 + 11^2 + 13^2 + 17^2$: The sum of the squares of the first seven prime numbers
4) $666 = 313 + 353$: The sum of two consecutive palindromic primes (numbers that read the same forward and backward).
5) $666 = 2x3x3x37$ and $6 + 6 + 6 = 2 + 3 + 3 + 3 + 7$. 666 is called a Smith number since the sum of its digits is equal to the sum of the digits of its prime factors.
6) $666^2 = 443556$ and $666^3 = 295408296$. Furthermore, $\left(4^2 + 4^2 + 3^2 + 5^2 + 5^2 + 6^2\right) +$ $\left(2 + 9 + 5 + 4 + 0 + 8 + 2 + 9 + 6\right) = 666$
7) 666 is made from the sequence 123456789 by insertion of one or more plus signs in two different ways: $1 + 2 + 3 + 4 + 567 + 89 = 666 = 123 + 456 + 78 + 9$
8) Likewise for 987654321: $666 = 9 + 87 + 6 + 543 + 21$.

Challenge: Are there other ways that you can make the number 666 from the sequences in 7) and 8) using only plus signs? Using both plus and minus signs?

Two is Equal to One! *AL*

What is wrong with the following demonstration that algebraically shows—without a doubt—that two is equal to one? *Hint*: Think back to the cardinal no-no of algebra!

Demonstration that Two Equals One

1. Set $x = y$
2. Multiply both sides by x: $x^2 = yx$
3. Subtract y^2 from both sides: $x^2 - y^2 = xy - y^2$
4. Factor both sides: $(x-y)(x+y) = y(x-y)$
5. Divide both sides by $x - y$: $x + y = y$
6. But $x = y$ by statement 1. Substituting, $2y = y$.
7. Dividing both sides by y, we have $2 = 1$. $\therefore$

Crossing Problems Old and New *LG*

Logic problems where several animals, objects, and/or people must cross over a river under a set of constraints have entertained and baffled puzzle solvers for many centuries. Below are two such problems. The first is at least 1000 years old, and urban legend has it that the second was a question on a Microsoft employment exam. Enjoy the two challenges!

Wolf, Goat, and Cabbage

A farmer and his goat, wolf, and cabbage come to a river that they wish to cross. There is a boat, but it only has room for two, and the farmer is the only one who can row. However, if the farmer leaves the shore in order to row, the goat will eat the cabbage, and the wolf will eat the goat.

Challenge: Devise a minimum number of crossings so that all concerned make it across the river alive and in one piece.

The "U2" Concert

"U2", the four-man Irish rock band, has a concert that starts in 17 minutes, and they all must cross a bridge in order to get there. All four men begin on the same side of the bridge. Your job is to devise a plan to help the group get to the other side on time. There are several constraints that complicate the crossing process: It is night and a flashlight must be used, but there is only one flashlight available. Any party that crosses—only 1 or 2 people allowed on the bridge at any given time—must have the flashlight with them. The flashlight must be walked back and forth; it cannot be thrown, teleported, etc. Each band member walks at a different speed, and a pair walking together must cross the bridge using the slower man's speed. Here are the four crossing times: Bono takes 1 minute to cross; Edge, 2 minutes to cross; Adam, 5 minutes to cross; and Larry, 10 minutes to cross.

Challenge: Devise the plan!

President Garfield and Pythagoras *GE, AL*

The Pythagorean Theorem was known in antiquity at least 1000 years before Pythagoras (circa 600 BC), the Greek mathematician who developed the first known proof. Proving the Pythagorean Theorem—especially constructing new proofs—has been a source of intellectual entertainment for many centuries. President Garfield constructed an original proof based on trapezoid properties while still a Congressman. Today, there are over 300 known proofs of the Pythagorean Theorem.

Challenge: Using the two figures below as your guide, can you reconstruct the associated unique proofs of the Pythagorean Theorem? Figure 1 was the one used by President Garfield and Figure 2 has its origins in ancient China.

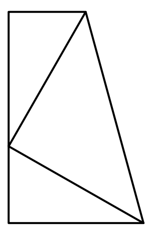


Figure 1

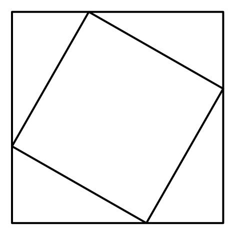
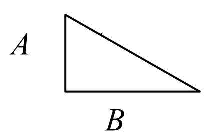


Figure 2



Statement of the Pythagorean Theorem

Let a right triangle have sides of lengths $A, B,$ and C where C is the hypotenuse. Then the following relationship holds:

$$C^2 = A^2 + B^2 .$$

A Potpourri of Powers *AR, CU*

After seeing the Pythagorean relationship between three squares, one might ask what other neat relationships exist amongst numbers, multiples, digits, and powers. Below is a connoisseur's sampling. Enjoy! *Challenge*: If you don't believe one of the statements below, then check it out.

1. $6^3 = 3^3 + 4^3 + 5^3$
2. $49 = 47 + 2$ and $94 = 47x2$
3. $371 = 3^3 + 7^3 + 1^3$ and $407 = 4^3 + 0^3 + 7^3$
4. $135 = 1^1 + 3^2 + 5^3$ and $175 = 1^1 + 7^2 + 5^3$
5. $169 = 13^2$ and $961 = 31^2$
6. $244 = 1^3 + 3^3 + 6^3$ and $136 = 2^3 + 4^3 + 4^3$
7. $499 = 497 + 2$ and $994 = 497x2$
8. $504 = 12x42 = 21x24$
9. $1634 = 1^4 + 6^4 + 3^4 + 4^4$ and $3435 = 3^3 + 4^4 + 3^3 + 5^5$
10. $2025 = 45^2$ and $20 + 25 = 45$
11. $4913 = 17^3$ and $4 + 9 + 1 + 3 = 17$
12. 9240 has 64 divisors! Can you find them all?
13. $54{,}748 = 5^5 + 4^5 + 7^5 + 4^5 + 8^5$
14. $321489 = 567^2$ Not counting the exponent 2, this equality uses each of the nine digits just once. The only other number that does this is 854.

✓

The 3 by 3 Magic Square *AR, CU*

Kids love magic squares, and magic squares are a great way to encourage a child to practice addition skills. Start them out on the 3 by 3 magic square shown below, which uses the numbers 1 through 9.

2	7	6
9	5	1
4	3	8

Explain to them that if we add all the numbers in any one row or column, the sum is always 15 (called the magic constant). The same is true for the numbers on the two diagonals. Two questions I like ask are, “Can you show this is true?” and “How many different ways can we make a sum of 15?” An interested fact is that we have records of 3 by 3 magic squares dating back to 1000 BC.

A cool challenge for the young math student: Add 10 to each number in the magic square above. Is the new square also a magic square? If so, what is the magic constant?

A Perfect 4 by 4 Magic Square *AR, CU*

Once kids have explored the 3 by 3 magic square, introduce them to the 4 by 4 magic square shown below, which uses the numbers 1 through 16 and has a magic constant of 34.

1	15	6	12
8	10	3	13
11	5	16	2
14	4	9	7

In this 4 by 4 (as in the 3 by 3) all rows, columns, and diagonals sum to the magic constant. But…that is only the beginning of the story. The four corners of the 4 by 4 square also sum to the number 34. Now, go ahead and examine the four corners of any size sub-square contained anywhere in the big square. These corners also sum to the number 34! A 4 by 4 magic square that has this amazing four-corner-summing property is also called a perfect square.

Challenge: In how many different ways can the number 34 be made? This ought to keep those kids—and their teacher—busy for a while! *Super Challenge*: Can you find any other geometric patterns of four numbers within this square which also sum to 34.

Kaprekar Teaches Subtraction (Part 1) *AR, CU*

Shri Dattathreya Ramachanda Kaprekar was an Indian mathematician who discovered a fascinating “number-crunching” process in the late 1940s that involved the use of three and four digit numbers. Today, Kaprekar’s Process is a wonderful tool (or game) that can aid in the building of subtraction skills.

Take any three-digit number whose digits are not all the same (222 is not OK, but 221 is OK). Rearrange the digits twice in order to make the largest and smallest numbers possible. Subtract the smaller number from the larger. Take the result and repeat the process. Let’s see what happens for three different three-digit numbers!

517	**263**	**949**
751-157=594	632-236=396	994-499=545
954-459=495	963-369=594	554-455=099
954-459=**495**	954-459=**495**	990-099=891
		981-189=792
		972-279=693
		963-369=594
		954-459=**495**

Two potential activities

*- Count the cycles to reach **495***

- Have a subtraction race

Notice that the cycle ends (or stalls) at the number **495** (called the Kaprekar constant) each time we run the process. Point: Remember to write two-digit results using three digits (i.e. 99 becomes 099 in the **949** cycle).

Challenge: Now, you try it. Then, let your kids try it, and watch them subtract!

Kaprekar Teaches Subtraction (Part 2) *AR, CU*

Kaprekar's process also works for four-digit numbers, where the Kaprekar constant is **6174**. So, let's run Kaprekar's process on the number 1947 (my birth year) which suggests several great subtraction activities for your classroom.

1947

9741-1479=8262

8622-2268=6354

6543-3456=3087

8730-0378=8352

8532-2358=6174

7641-1467=**6174**

Potential Classroom Activities:

1) Run the Kaprekar process for your birth year
2) Count the cycles to reach 6174
3) Now let each student run the Kaprekar process for their own birth year and count the cycles to 6174
4) Which student has the most cycles?
5) Which student has the least cycles?

Additional Challenge: There is also a Kaprekar constant for two-digit numbers. Can you find it?

Perfect Numbers Big and Small *AR, CU*

Do you have little angels your classroom? Probably not! But, there are numbers which are called perfect. A perfect number is simply a number that is equal to the sum of all of its proper divisors (a proper divisor of a number is any divisor smaller than the number itself). The first perfect number is **6** since 6=1+2+3 (all divisors smaller than 6). To show **28** is perfect, just note that 28=1+2+4+7+14. Perfect numbers are quite rare—such as perfect children—and grow rapidly in size. Below is a list of the first seven perfect numbers.

6: known to the ancient Greeks

28: known to the ancient Greeks

496: known to the ancient Greeks

8128: known to the ancient Greeks

33550336: recorded in medieval manuscript

8589869056: Cataldi discovered in 1588

137438691328: Also discovered by Cataldi in 1588

Challenge: Can you or your class show that **496** is a perfect number? Do you dare examine **8128**!

Are You Abundant or Deficient? *AR*

Any number, such as your birth year, has proper divisors. If all proper divisors of a number sum to more than the number itself, the original number is called abundant. Deficient numbers are where the opposite is true (sum to less than the number itself). All prime numbers are deficient.

Challenge: Have each member of your class take their birth year and see if it is abundant or deficient. Who has the most abundant number (exceeds the birth year by the greatest amount)? Abundant or not, I am feeling pretty deficient when I have the flu! How about you?

Friendly Pairs *AR, CU*

A pair of numbers is called friendly if each number in the pair is the sum of the proper divisors for the other number. The first and smallest friendly pair is **220** and **284**, discovered by the Greeks. Take a look:

220=1+2+4+71+142 (all of the proper divisors of 284)

284=1+2+4+5+10+11+20+22+44+55+110
(all of the proper divisors of 220)

Three other pairs of friendly numbers are shown below. Today, over 1000 pairs of friendly numbers are known.

1184 & **1210**: discovered by Paganini in 1866 at age 16

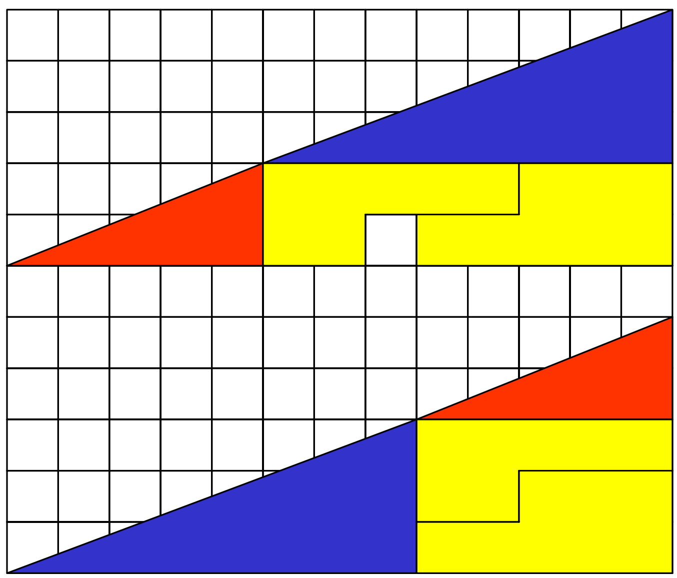
17,163 & **18,416**: discovered by Fermat in 1636

9,363,584 & **9,437,056**: discovered by Descartes in 1638

Challenge: Can you show that **1184** and **1210** are friendly?

One Little Mysterious Area *GE*

We are now going to return to geometry and explore a little mystery! Each of the two triangular arrangements shown below uses the same four pieces. But alas! One of the areas has a little square missing. Where did it go?



The Camel and the Bananas *LG, AL*

A camel used to transport bananas must travel 1000 miles across a desert to reach customers living in an exotic city. At any given time, the camel can carry up to 1000 bananas and must eat one banana for every mile it walks.

Challenge: Assuming an initial stock of 3000 bananas, what is the maximum number of bananas that the camel can transport across the desert and into the eager hands of waiting customers who live in the exotic city?

Word Morphing with Lewis Carroll *WP, LA*

Lewis Carroll—mathematician, teacher, and author of Alice in Wonderland—invented a marvelous word game in the 1870s that he called "Doublets". Nowadays, I'll call it word morphing. Here is how it goes: Take two words having the same number of letters, say **cat** and **dog**. Can you transform (morph) the **cat** into a **dog** by changing only one letter at a time where each intermediate form is a bona-fide word in the English language? **Cat** and **dog** are easy. Consider the sequence—**cat**, **cot**, **dot**, and **dog**—which solves the problem quite nicely. Again, every word in the sequence (Carroll called this a chain) must be an English word, and the player can only change one letter at a time. Also, the original rules prohibit switching letters within a word. Here is another example; to turn **warm** into **cold**, construct the sequence: **warm**, **ward**, **card**, **cord**, and **cold**. One can have fun anywhere and almost anytime with Lewis Carroll's wonderful little word game!

Challenge: Back in Carroll's day, nobody could take the **horse** to the **field**. I am now told that English words are available that can make this chain happen—your move!

The Three Bears *WP, LA*

Which of the following sentences, if any, bears errors?

1) No bear bare should bare a burden.
2) No bear bare should bear a burden.
3) No bare bear should bear a burden.

Prime Time *AR, AL*

A prime number is an integer (or counting number) that has no proper divisors other than one. Hence, by definition, all prime numbers are deficient. Even so, mathematicians still love prime numbers and have been fascinated by their properties for many years. Below is a table showing the prime numbers less than 100.

2	3	5	7	11
13	17	19	23	29
31	37	41	43	47
53	59	61	67	71
73	79	83	89	97

The mathematician Leonard Euler (1707-1783) discovered the following simple formula that generates prime numbers P for all counting numbers x starting with 0 and continuing through 39:

$$P = x^2 + x + 41.$$

For example, when $x = 5$, we have that $P = 5^2 + 5 + 41 = 71$,a prime number!

The above formula can be used in at least two skill-building activities suitable for beginning algebra students.

Challenge: Use Euler's formula above to generate all 39 prime numbers, checking each to see if it really is prime. *Super Challenge*: Set P equal to each prime number in the table above and solve for the x that makes it so!

The Why of Signs *AL*

Have you ever wondered why a negative number times a negative number is a positive number? The following is a demonstration that I have used for many years to show this.

In the first column below, as the second number steps from 5 to –5, the product decreases by 4 each time. Hence, it makes sense that zero times a number should be zero, and a positive number times a negative number should be a negative number. In the second column, as the second number steps from 4 to –5, the product increases by 5 each time leading to the natural conclusion that a negative number times a negative number should be a positive number. In both columns, the overall pattern shows without a doubt—it is the way signs ought to be!

	Now reverse the –5 and 4
4x5=20	
4x4=16	(-5)x4=-20
4x3=12	(-5)x3=-15
4x2=8	(-5)x2=-10
4x1=4	(-5)x1=-5
4x0=0	(-5)x0=0
4x(-1)=-4	(-5)x(-1)=5
4x(-2)=-8	(-5)x(-2)=10
4x(-3)=-12	(-5)x(-3)=15
4x(-4)=-16	(-5)x(-4)=20
4x(-5)=-20	(-5)x(-5)=25

Those Powers to Be *AL*

We can use a progressing pattern similar to the one in "The Why of Signs" to illustrate the way exponents of the negative and zero kind ought to be.

$$64 = 2^6$$
$$32 = 2^5$$
$$16 = 2^4$$
$$8 = 2^3$$
$$4 = 2^2$$
$$2 = 2^1$$

Notice that for every step, the number on the left side of the equals sign is the previous number divided by two while the corresponding exponent is the previous exponent decreased by one. This sequence leads to the following natural interpretation of negative and zero exponents.

$$1 = 2^0$$
$$\tfrac{1}{2} = 2^{-1}$$
$$\tfrac{1}{4} = 2^{-2}$$
$$\tfrac{1}{8} = 2^{-3}$$
$$\tfrac{1}{16} = 2^{-4}$$
$$\tfrac{1}{32} = 2^{-5}$$
$$\tfrac{1}{64} = 2^{-6}$$

Two facts follow: $a^0 = 1$ and $\dfrac{1}{a^n} = a^{-n}$ *for any* $a \neq 0$.

Here Lies Old Diophantus *AL*

The Greek Mathematician Diophantus of Alexander (born about 200 AD) is considered by many historians to be the father of algebra. He wrote a book called Arithmetica, the earliest written record containing variables, algebraic equations, and solutions. There is an epithet for Diophantus (published about 500 AD) describing his life in terms of a riddle: "This tomb holds Diophantus. Ah, how great a marvel! The tomb tells scientifically the measure of his life. Zeus granted him to be a boy for one-sixth of his life, and adding a twelfth part to this, Zeus clothed his cheeks with down. He lit him the light of wedlock after a seventh part, and five years after his marriage Zeus gave him a son. Alas, late-born wretched child! After obtaining the measure of half his father's life, chill Fate took him. After, consoling his grief by the study of numbers for four years, Diophantus ended his life."

Challenge: From this riddle, can you determine how old Diophantus was when he died?

Young Gauss Stuns His Teacher *AR, EA*

Carl Gauss (1777-1855) is considered by many to be one of the greatest mathematicians of all time. Legend has it that he entered school at the age of 5 and stunned his teacher who gave him a tedious problem to solve, thinking it would take the lad an hour or more. Here is the problem: add the counting numbers 1 through 100. The answer is 5050, and Gauss had determined it within one minute!

Challenge: How did young Gauss solve the problem so quickly? Can you extend his technique to add the counting numbers 1 though 2003? *Super Challenge*: Can you use his technique to determine the value of the magic constant for a 5x5 (see title page of this section) magic square? A 7x7?

One Dollar Please *LG*

The following logic puzzle is very old. It always seems to challenge each new generation of thinkers as the story line gets updated to fit changing times. Three men stayed for one night in a motel, all three sharing the same room. They checked out the next morning, the bill for the night coming to \$25.00. Each man gave the motel clerk a \$10.00 bill and told him to keep \$2.00 of the change as a tip. The clerk gave \$1.00 in change back to each of the three men. A quick reckoning has the night costing \$27.00 plus a \$2.00 tip.

Challenge: Where did the other dollar go?

Four Fours Puzzle *AR, AL*

Challenge: Create all the counting numbers 0 through 100 using mathematical equalities having exactly four 4s and no other numerals on the left hand side.

Two examples are $4x4x4-4=60$ *and* $4 \div 4 + 4! + \sqrt{4} = 27$.

My Problem with Ice Cream *AR, AL*

My problem with ice cream is that I love it, and I always have! The problem below is for all ice-cream lovers. And, if you are a true ice-cream lover, I can imagine you saying, “Not a problem!” Now, the local ice-cream parlor sells monster once-in-a-lifetime sundaes for those very special occasions. A customer is allowed to pick from three flavors: chewy double chocolate crunch (\$1.00 per scoop), multi-berry ambrosia (\$1.60 per scoop), and Aegean vanilla (\$.80 per scoop). A monster sundae costs \$20.00, \$16.00 for 15 scoops of ice cream and an additional \$4.00 for an assortment of delectable toppings.

Challenge: In how many different ways can I order my monthly—oops—treat? What are they? Note: the parlor will not serve partial scoops.

Ben Franklin’s 8 by 8 Magic Square *AR, CU*

Benjamin Franklin loved magic squares. In 1769, he invented the 8 by 8 magic square shown below using the counting numbers 1 through 64. Now Ben’s square has a little problem: if you sum the numbers for each row, column, and diagonal (18 different ways altogether), the totals miss the mark in two cases. I guess he had a hard time fitting it all in—perhaps leading to some very long and sleepless nights!

Challenge: Can you discover the two inconsistencies in old Ben’s square? *Hint*: you may want to review your addition skills before you start.

52	61	04	13	20	29	36	45
14	03	62	51	46	35	30	19
53	60	05	12	21	28	37	44
11	06	59	54	43	38	27	22
55	58	07	10	23	26	39	42
09	08	57	56	41	40	25	24
50	63	02	15	18	31	34	47
16	01	64	49	48	33	32	17

Super Challenge: Is there such a thing as a 2 by 2 magic square using the counting numbers 1, 2, 3, and 4? Why or why not?

Finding Your Palindrome *AR, CU*

A palindrome is a number that reads the same forwards and backwards. 1374731 is a palindrome and so is 1551. Take any number and reverse its digits and add the new number to the original number. Repeat this process. Eventually, this reverse-and-add process has a good chance of producing a palindrome. Out of all counting numbers less than 100,000, only 5996 numbers fail to produce palindromes via this method. 196 is the smallest. Other numbers may require quite a few steps in order to finally reach a palindrome (e.g. 187 takes 23 steps).

Our two sons were born in 1973 and 1980. Let's use the digit reversal process to chase down their palindromes. Fortunately, both of our sons have one, and both birth years take five steps to produce a palindrome.

1973		1980	
3791		0891	
5764 *step 1*			2871 step 1
4675		1728	
10439 *step 2*			4653 step 2
93401		3564	
103840 *step 3*			8217 step 3
048301		7128	
152141 *step 4*			15345 step 4
141251		54351	
293392 *palindrome*			69696 *palindrome*

Challenge: Does your birth year have a palindrome associated with it? Can you find it? How about other members of your family?

Magic or Algebra? *AR, AL, CU*

The following little bit of numerical mystery is great fun for those in elementary school. Those a little older may want to ponder whether this little bit of jiggery-pokery is really magic or has its secrets in algebra…

1. Pick a number from 1 to 9
2. Multiply the number by two
3. Add 5
4. Multiply the result by 50
5. Answer the following question yes or no: have you had your birthday this year?
6. Add the magic number as given from the table below!
7. Subtract the four-digit year that you were born

You should end up with a three-digit number. The first digit should be the number that you originally picked and the last two digits should be your age! Wow!

For the year	If no, add	If yes, add
2002	1751	1752
2003	1752	1753
2004	1753	1754
2005	1754	1755
2006	1755	1756

There are Three Levels of Challenges:

1) Elementary school: make it work and impress all of your friends as you perform this trick!
2) Middle school: figure out why it works!
3) High school: make up your own piece of numerical magic following this general pattern and impress your less-mathematical friends!

Word Squares *WP, LA*

Word squares, which were very popular throughout the 1800s, are the language equivalent of magic squares and the forerunners to the modern crossword puzzle. Below are five word squares of various sizes.

2x2	*3x3*	*4x4*	*5x5*	*6x6*
ON	BAG	LANE	STUNG	CIRCLE
NO	APE	AREA	TENOR	ICARUS
	GET	NEAR	UNTIE	RAREST
		EARS	NOISE	CREATE
			GREET	LUSTRE
				ESTEEM

As shown, each square is composed of words of equal length that read in exactly the same way both horizontally and vertically. Diagonals do not have to be words. The 6x6 above is famous because when it first appeared in 1859, it claimed—tongue in cheek—to have solved the problem of "squaring the circle" (see note below). 7x7, 8x8, and 9x9 word squares are in existence today—but no 10x10!

Challenge: Try to construct a word square consisting of words unique to your family, town, favorite sports team, etc. in such a way that all the words support the same general idea. *Super Challenge*: Construct a 10x10 word square and become famous! Gain entry into Ripley's Believe it or Not!

The problem of squaring the circle is an ancient Greek geometric problem that requires the solver to construct a square whose area is equal to that of a given circle using only a compass and straight edge. This problem has since been proven unsolvable under the two classical constraints—compass and straightedge—as given.

Narcissistic Numbers *AR, CU*

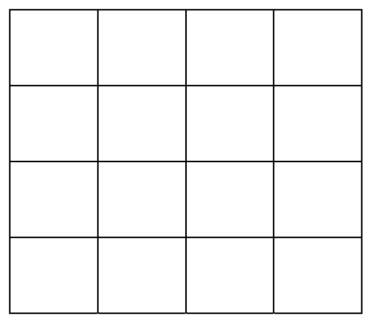
According to Greek mythology, Narcissus fell in love with his own image while looking into a pool of water. He subsequently turned into a flower that "bears" his name. Narcissistic numbers are numbers whose own digits can be used to recreate themselves via established rules of arithmetic. A lovely sampling is below—all waiting to be verified by the willing student! Also included are three examples of "narcissistic pairs".

1. $3435 = 3^3 + 4^4 + 3^3 + 5^5$
2. $127 = -1 + 2^7$
3. $598 = 5^1 + 9^2 + 8^3$
4. $3125 = (3^1 + 2)^5$
5. $1676 = 1^1 + 6^2 + 7^3 + 6^4 = 1^5 + 6^4 + 7^3 + 6^2$
6. $759375 = (7 - 5 + 9 - 3 + 7)^5$
7. $2592 = 2^5 \cdot 9^2$
8. $1233 = 12^2 + 33^2$
9. $990100 = 990^2 + 100^2$
10. $94122353 = 9412^2 + 2353^2$
11. $2646798 = 2^1 + 6^2 + 4^3 + 6^4 + 7^5 + 9^6 + 8^7$
12. $2427 = 2^1 + 4^2 + 2^3 + 7^4$
13. $24739 = 2^4 \cdot 7! \cdot 3^9$
14. $3869 = 62^2 + 05^2$ & $6205 = 38^2 + 69^2$ *pair*
15. $5965 = 77^2 + 06^2$ & $7706 = 59^2 + 65^2$ *pair*
16. $244 = 1^3 + 3^3 + 6^3$ & $136 = 2^3 + 4^3 + 4^3$ *pair*
17. $343 = (3 + 4)^3$
18. $221859 = 22^3 + 18^3 + 59^3$
19. $416768 = 768^2 - 416^2$
20. $3468 = 68^2 - 34^2$

Coloring the Grid *LG, GE*

We keep coming back to squares in this little volume. This time, the object is to color the 4 by 4 grid shown below where 4 of the little squares are to be blue, 3 are to be green, 3 are to be white, 3 are to be yellow, and 3 are to be red. Oh yes, there is one additional requirement.

Challenge: Color the grid so that no color appears more than once in any horizontal, vertical, or diagonal line.



The 100 Puzzle *LG, AR*

The 100 Puzzle is a very old favorite which can be used in the middle grades as an arithmetic enrichment exercise. Here is how it goes. First, write the digits one through nine in natural order. Now, without moving any of the nine digits, insert arithmetic signs and/or parenthesis so that the digits total to 100. Dudeney, one of the greatest puzzles creators of all times, claimed that

$$1+2+3+4+5+6+7+(8x9)=100$$

was the most common solution. He came up with many solutions during his lifetime including this favorite:

$$123-45-67+89=100.$$

Dudeney liked this particular solution because it minimized the number of arithmetic signs.

Yet another solution is $12+3-4+5+67+8+9=100$.

Four challenges for your math class.

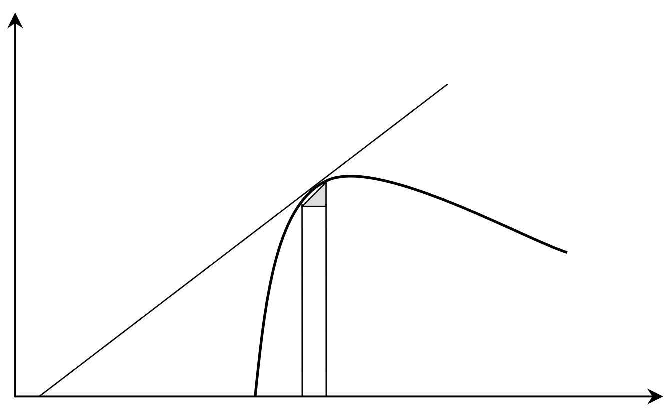
1) Conduct a contest to see who can come up with the most solutions.
2) Conduct a contest to see who can come up with a solution having a minimum number of signs.
3) Reverse the digits one through nine (writing them in decreasing order) and play The Reverse 100 Puzzle.

   One solution is $98+7-6+5-4+3-2-1=100$.

4) Write the nine digits in random order and play 1) and 2) above.

Some Wiz Stuff

Calculus and Statistics Gems
for Math-Inclined Parents

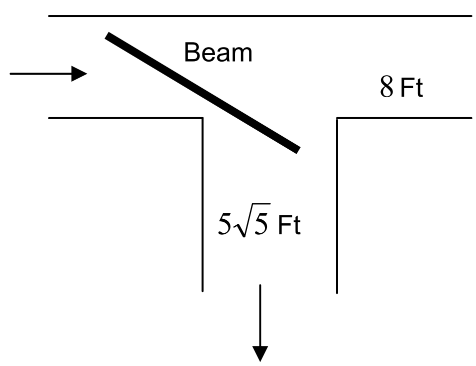


Barrow's Diagram

The above figure was originally created by Isaac Barrow (1630-1677) who was a geometer, first occupier of the Lucasian chair at Cambridge, and a teacher/mentor to Sir Isaac Newton.

The Famous Girder Problem *Calculus*

The problem below started to appear in calculus texts circa 1900. My father first experienced it in 1930 as an engineering student, and I first encountered it in the winter of 1966. It still appears in modern calculus textbooks disguised—and somewhat watered down—as an geometric optimization problem. The girder problem is famous because of the way it thoroughly integrates the principles of plane geometry, algebra, and differential calculus. My experience as a teacher has been that "many try, but few succeed. " Will you? Have fun!



The problem and the challenge: Two people at a construction site are rolling steel beams down a corridor 8 feet wide into a second corridor $5\sqrt{5}$ feet wide and perpendicular to the first corridor. What is the length of the longest girder that can be rolled from the first corridor into the second corridor and continued on its journey in the construction site? Assume the beam is of negligible thickness.

One Mean Derivative *Calculus*

There is an old maxim of mathematics that says, "You really learn your algebra when you take calculus; and if you don't, calculus will take you." What follows is the first calculus problem in the book. But wait, is it more of an algebra problem. You decide!

Challenge: For the function $f(x)=\dfrac{1}{\sqrt{x^3}}$, find the derivative $f'(x)$ by appealing to the basic definition for $f'(x)$ which embodies the limit concept: $f'(x)=\lim_{h\to 0}\left[\dfrac{f(x+h)-f(x)}{h}\right]$.

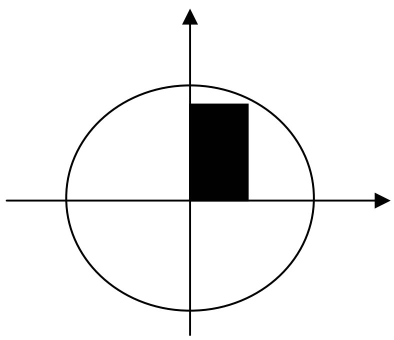
Additional Challenge: Brush off some dusty neurons and see if you can determine the equation of the line tangent to the graph of $f(x)$ at $x=4$. Also, determine the equation of the line normal to the graph of $f(x)$ at $x=4$.

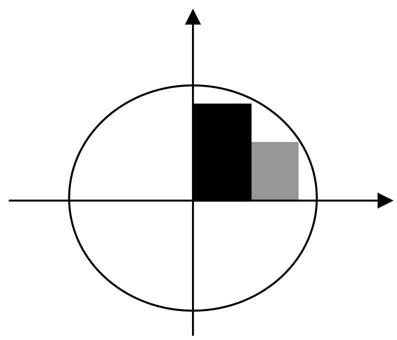
No Calculators Allowed! *Calculus*

Use the techniques of differential calculus to show that $e^{\pi}>\pi^{e}$.

A Mathematician's Desert *Calculus*

A frequent problem in first term calculus is to find the area of the largest rectangle that can be inscribed inside the first-quadrant portion of the unit circle. See the figure on the left below. For those of you who haven't worked with calculus for a while, I suggest that this well-known problem (which is solved using single-variable differential calculus) be your warm-up.





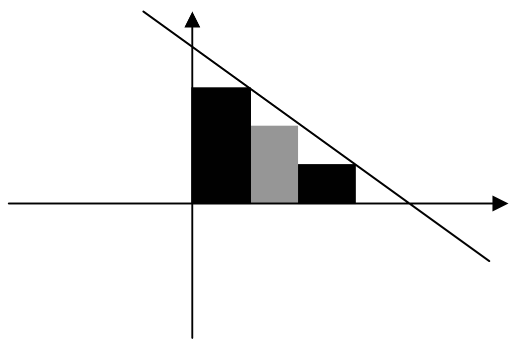
Challenge: It doesn't take much of an expansion to turn the above problem into a connoisseur's absolute delight consisting of two independent variables, partial derivatives, and subtle observations—not to mention a king-sized scoop of algebra. Here it comes! The figure on the right is joined to the following question: Find the dimensions for each of the two rectangles inscribed in the unit circle as shown that maximizes the total combined area of both rectangles.

Note: An Air Force Captain first introduced me to this problem in 1981 after experiencing it on a Ph.D. qualifying examination the week before. Thanks Joe for a superb treat!

Triple Play *Calculus*

Calculus is always more fun when two or three variables are in the game. The problem below is similar to the one presented in “A Mathematician’s Desert”, but we have added one more variable to the lineup!

Challenge: Consider the figure below where the equation of the line segment is given by $y = 1 - x$ where $0 \leq x \leq 1$.



Find the dimensions for each of the three rectangles inscribed in the triangular region as shown that maximizes the total combined area of the threesome.

$$\int_a^b f(x)dx$$

Calling all Data Lovers *Statistics*

There is a small, well-kept cemetery close to where I live—the final resting-place for 180 (last count) Catholic priests and brothers. Below is the data summary from all 180 headstones. Each four-digit entry is the year of death (with the 19 omitted) and age at death. For example, the first entry 6245 codes a death in 1962 at age 45.

6245, 6286, 6338, 6346, 6383, 6393, 6462, 6464, 6475, 6488, 6557, 6671, 6679, 6682, 6763, 6784, 6832, 6839, 6846, 6854, 6866, 6876, 6877, 6877, 6883, 6884, 6952, 6957, 6984, 7033, 7059, 7065, 7072, 7079, 7086, 7087, 7143, 7167, 7168, 7176, 7182, 7189, 7236, 7252, 7261, 7275, 7287, 7288, 7356, 7369, 7462, 7467, 7468, 7471, 7474, 7478, 7550, 7666, 7667, 7667, 7676, 7678, 7682, 7690, 7741, 7764, 7774, 7775, 7784, 7967, 7968, 7969, 7972, 7974, 7977, 7990, 8082, 8084, 8164, 8167, 8170, 8172, 8182, 8182, 8183, 8184, 8191, 8246, 8259, 8266, 8275, 8276, 8286, 8290, 8294, 8373, 8376, 8378, 8385, 8468, 8474, 8477, 8479, 8480, 8567, 8569, 8569, 8569, 8570, 8579, 8580, 8584, 8666, 8672, 8673, 8678, 8681, 8769, 8769, 8774, 8781, 8790, 8864, 8870, 8878, 8888, 8889, 8954, 8973, 8973, 8979, 8990, 8993, 9067, 9082, 9083, 9088, 9149, 9155, 9171, 9176, 9181, 9183, 9260, 9276, 9294, 9357, 9368, 9380, 9380, 9385, 9390, 9392, 9467, 9471, 9483, 9483, 9484, 9486, 9486, 9568, 9580, 9583, 9583, 9584, 9658, 9662, 9677, 9677, 9678, 9679, 9680, 9682, 9684, 9686, 9689, 9693, 9781, 9788, 9797: **R.I.P.**

Challenge: can you argue a case for increasing male longevity in the United States from this particular sample? Why or why not? Let statistics (or sadistics as some of my students like to say) be your guide.

Answers to Selected Problems *by page number*

Page 12

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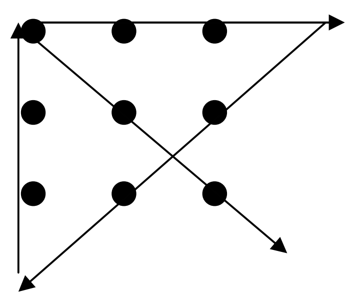
Page 14

1)



2) 30 squares total

Page 17: Step 5, division by 0

Page 18: F, W, G, C: F, G cross; F comes back; F, W cross; F,G come back; F, C cross; F comes back; F, G cross for the last time.

U2: E, B cross (2 minutes); E comes back (2 minute); A, L cross (10 minutes), B comes back (1 minute), E, B cross for the second time (2 minutes). Times total 17 minutes.

Page 21: Yes. The magic constant is 45.

Page 27:

1) Hint, are the two figures really triangles?
2) 533 1/3 bananas

Page 32: 84 years old

Page 33:

Stage	3 Men	Hotel	Bell	Sum
1) Before entering	30	0	0	30
2) Front Desk	0	30	0	30
3) Send Back	0	25	5	30
4) Distribution	3	25	2	30

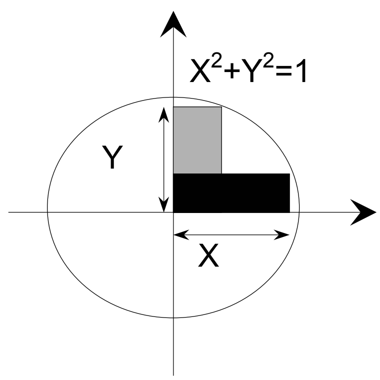
Page 39 :

G	W	B	Y
B	Y	R	G
R	G	W	B
W	B	Y	R

Page 42 : Exactly 27 feet

Page 43: Hint, analyze $f(x) = e^x - x^e$ on $0 \le x \le \infty$.

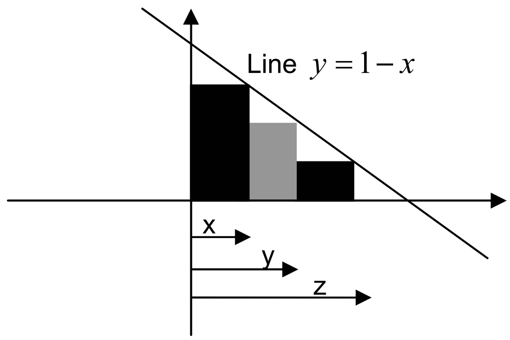
Page44: See diagram below. Note the definitions of X and Y!



Let $A(x, y) = x\sqrt{1-x^2} + (y - \sqrt{1-x^2})\sqrt{1-y^2}$.

Set $\frac{\partial A}{\partial X} = \frac{\partial A}{\partial y} = 0 \Rightarrow X = Y = .85$ (rounded to two decimals).

Page 45: Set up the function for total area as shown below and optimize for $0 \le x \le y \le z \le 1$. Have fun!



$$A(x, y, z) = x(1-x) + (y-x)(1-y) + (z-y)(1-z)$$

Page 46: Just one example of what is possible